Effective actions for 0+1 dimensional scalar QED and its SUSY generalization at $T\neq 0$

J. Barcelos-Neto* and Ashok Das
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
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We compute the effective actions for the 0+1 dimensional scalar field interacting with an Abelian gauge background, as well as for its supersymmetric generalization at finite temperature. [S0556-2821(99)02008-1]

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The effective action for 0+1 dimensional fermions interacting with a background Abelian gauge field, at finite temperature, has received much attention during the past year [1–3]. In this Brief Report, we present the finite temperature effective actions for scalar fields interacting with an Abelian gauge background as well as for its supersymmetric extension.

Let us consider the interacting scalar theory described by the Lagrangian

$$L=(D_t\phi_j)^*(D_t\phi_j)-m^2\phi_j^*\phi_j,\quad j=1,2,\ldots,N_f \tag{1}$$

where

$$D_t\phi_j=\partial_t\phi_j+iA\phi_j. \tag{2}$$

Unlike the theory of massive fermions [1], this theory is, in fact, invariant under charge conjugation and, consequently, the effective theory can only contain terms involving an even number of photons. The simplest, of course, is the two point function and involves two distinct Feynman diagrams. At zero temperature, this gives

$$\begin{aligned} iI^{(2)}(p)=\int\frac{dk}{2\pi}\bigg[&-\frac{2}{k^2-m^2+i\epsilon}\\ &+\frac{(2k+p)^2}{(k^2-m^2+i\epsilon)[(k+p)^2-m^2+i\epsilon]}\bigg], \end{aligned} \tag{3}$$

$$=0,$$

implying that there is no quadratic term (in A) in the effective action.

As we go to higher point functions, the number of graphs increases rapidly and a diagrammatic evaluation becomes complicated. Consequently, we follow an alternate procedure. Let us note that the effective action obtained by integrating out the scalar fields has the form (properly normalized)

$$\begin{aligned}\Gamma&=iN_f\log\frac{\det(-D_t^2-m^2)}{\det(-\partial_t^2-m^2)}\\ &=iN_f\left[\log\frac{\det(iD_t-m)}{\det(i\partial_t-m)}+\log\frac{\det(iD_t+m)}{\det(i\partial_t+m)}\right]\\ &=\tilde{\Gamma}(m)+\tilde{\Gamma}(-m),\end{aligned} \tag{4}$$

where $\tilde{\Gamma}(m)$ is the effective action obtained from the much simpler, first order scalar Lagrangian of the form

$$\tilde{L}=\tilde{\phi}_j^*(iD_t-m)\tilde{\phi}_j. \tag{5}$$

Calculationally, this is much simpler. In fact, at zero temperature, this would lead to the same effective action as from the fermionic theory [1–3] (except for the negative sign associated with the fermion loop)

$$\tilde{\Gamma}(m)=-\frac{N_f}{2}\mathrm{sgn}(m)\int dtA(t). \tag{6}$$

It follows from this that the effective action (4)

$$\Gamma=\tilde{\Gamma}(m)+\tilde{\Gamma}(-m)=0. \tag{7}$$

Namely, there is no radiative correction to the theory in Eq. (1) at zero temperature.

The evaluation of the effective action, at finite temperature, also follows in a straightforward manner. Let us consider the theory in Eq. (5) (with $m>0$ for simplicity) and note that because of the bosonic nature of the fields, the propagator at finite temperature has the form [4]

$$\tilde{G}(p)=\frac{i}{p-m+i\epsilon}+2\pi n_B(m)\delta(p-m) \tag{8}$$

with ($\beta=1/kT$)

$$n_B(m)=\frac{1}{e^{\beta m}-1}. \tag{9}$$

The propagator in coordinate space has the form

$$\tilde{G}(t)=[\theta(t)+n_B(m)]e^{-imt}. \tag{10}$$

The one-point function is easy to evaluate:

*Permanent address: Instituto de Física, Universidade Federal do Rio de Janeiro, RJ 21945-970, Brazil.

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$$i\widetilde{I}^{(1)}(t)=-iN_f[\theta(0)+n_B(m)]$$
$$=-\frac{iN_f}{2}\coth\frac{\beta m}{2}. \tag{11}$$

This, of course, reduces to the one-point function from Eq. (6) at zero temperature (for $m>0$). Furthermore, following [2], we can show that in this case, the recursion relation between amplitudes has exactly the same form as in the fermionic theory

$$\frac{\partial\widetilde{I}^{(N)}}{\partial m}=-i\beta(N+1)\widetilde{I}^{(N+1)}, \tag{12}$$

so that all the amplitudes are related to the one-point amplitude recursively. Thus, in this case, the effective action takes the form $[a=\int dtA(t)]$

$$\widetilde{\Gamma}(m)=-i\sum_{N=1}^{\infty}a^N(i\widetilde{I}^{(N)})$$
$$=\frac{i\beta N_f}{2}\sum_{N=1}^{\infty}\frac{1}{N!}\left(\frac{ia}{\beta}\right)^N\frac{\partial^{N-1}}{\partial m^{N-1}}\coth\frac{\beta m}{2}$$
$$=iN_f\log\left\{\cos\frac{a}{2}+i\coth\frac{\beta m}{2}\sin\frac{a}{2}\right\}. \tag{13}$$

Consequently, the effective action for the interacting scalar theory in Eq. (1) has the form [see Eq. (4)]

$$\Gamma=\widetilde{\Gamma}(m)+\widetilde{\Gamma}(-m)$$
$$=iN_f\log\left\{\cos^2\frac{a}{2}+\coth^2\frac{\beta m}{2}\sin^2\frac{a}{2}\right\}. \tag{14}$$

This can be easily seen to reduce to the zero temperature result of Eq. (7) and is invariant under the large gauge transformation $a\rightarrow a+2\pi N$ as well, for any number of flavors, N_f.

Let us next consider the supersymmetric (SUSY) generalization of Eq. (1),

$$L_{\rm SUSY}=(D_t\phi_j)^*(D_t\phi_j)-m^2\phi_j^*\phi_j+\overline{\psi}_j(iD_t-m)\psi_j. \tag{15}$$

The supersymmetric multiplet, (ϕ_j,ψ_j), is interacting with a background, Abelian gauge field. There is no photino in this theory and yet, because of the simplicity of $0+1$ dimension, it can be easily checked that this Lagrangian is invariant under the supersymmetry transformations

$$\delta\phi_j=\epsilon\psi_j,$$
$$\delta\psi_j=-i(D_t\phi_j)\epsilon,$$
$$\delta\lambda=A\epsilon, \tag{16}$$
$$\delta A=-i\epsilon\partial_t\lambda,$$

where we assume that a Majorana fermion is a real fermion, $\epsilon^*=\epsilon$ and $\overline{\psi}=\psi^*$. The transformations of the scalar and the charged fermion, in Eq. (16), are the conventional ones whereas those of the photon and the photino look unconventional. This is primarily because in $0+1$ dimension, the photon has no kinetic energy term and is like an auxiliary field and that a fermion with an auxiliary field can describe a supersymmetric multiplet in lower dimensions [5,6] (the conventional SUSY transformation would imply that the photino does not transform since the field strength associated with the gauge field vanishes). In fact, it is worth noting that a Chern-Simons term in $0+1$ dimension would be automatically supersymmetric under such a transformation (in fact, the Chern-Simons term can be supersymmetric only if A transforms like an auxiliary field). It is worth noting here that even though the transformation of A in Eq. (16) appears like a gauge transformation [be it with a fermionic parameter as in a Becchi-Rouet-Stora-Tyutin (BRST) transformation], it is because the auxiliary fields transform as a total derivative under a supersymmetry transformation and the photon, in this theory, is an auxiliary field. One can check explicitly that the transformations of (A,λ), indeed, satisfy the supersymmetry algebra and not the Abelian algebra of the gauge symmetry. The theory in Eq. (15) is, of course, not the most general supersymmetric interacting theory involving ϕ_j,ψ_j,A,λ. For example, it is easy to check that

$$L=(D_t\phi_j)^*(D_t\phi_j)-m^2\phi_j^*\phi_j+\overline{\psi}_j(iD_t-m)\psi_j$$
$$+i\lambda\dot{\lambda}+\lambda(\psi_j\phi_j^*-\overline{\psi}_j\phi_j)-\frac{1}{4}(\phi_j^*\phi_j)^2 \tag{17}$$

is also invariant under the transformations (16) on-shell. It is, however, not clear if this theory is soluble because of the quartic scalar interactions. On the other hand, it is interesting that the simple theory in Eq. (15) is already invariant under supersymmetry and is soluble.

The effective action for Eq. (15), of course, follows from Eq. (14) as well as from earlier work [1] and has the form

$$\Gamma_{\rm SUSY}=iN_f\left[\log\left\{\cos^2\frac{a}{2}+\coth^2\frac{\beta m}{2}\sin^2\frac{a}{2}\right\}\right.$$
$$\left.-\log\left\{\cos\frac{a}{2}+i\tanh\frac{\beta m}{2}\sin\frac{a}{2}\right\}\right]. \tag{18}$$

Furthermore, this action, in addition to being invariant under large gauge transformations, is also invariant under the supersymmetry transformations, Eq. (16). In fact, it is interesting to note that a SUSY transformation that leaves the Chern-Simons action invariant would also automatically be a symmetry of any nonextensive function of it. This behavior is quite distinct from the conventional expectation [7] that supersymmetry is broken at finite temperature.

In closing, we would like to add that the effective theory in Eq. (18) is purely bosonic and yet is supersymmetric. This is interesting and is complementary to the work in [8]. However, one can also trivially add a fermionic term (to give the theory a conventional form) to our starting theory (15) without changing any of the conclusions above simply as follows. Let

$$L' = \frac{i}{2}(\lambda + \xi)(\dot{\lambda} + \dot{\xi}) + \frac{1}{2}(A + \dot{\theta})^2, \tag{19}$$

where (θ,ξ) represent a Stuckelberg supermultiplet transforming under supersymmetry as (the Stuckelberg field θ transforms under a gauge transformation as $\delta_{\text{gauge}}\theta = -\alpha$, to maintain gauge invariance, where α is the parameter of gauge transformation)

$$\delta\theta = -i\epsilon\xi, \qquad \delta\xi = \epsilon\dot{\theta}. \tag{20}$$

These transformations also satisfy the supersymmetry algebra and the Lagrangian in Eq. (19) is invariant under the transformations of Eqs. (16) and (20). Adding this to the Lagrangian in Eq. (15), does not change the calculation of the effective action which would now be a sum of Eq. (18) and the action coming from Eq. (19).

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